# Theoretical Calculations for Verification of Rq

We know that the tool can be adjusted with a basic calibration multiplier and baseline offset to match any AFM measurements for Ra. We further know that the Rp measurements will vary between the AFM and THoT tool to a high degree depending on how the laser beam passes over the defect.

However, the measurement of Rq will vary between the instruments depending on the wavelength sensitivity. We know that the typical AFM frequency spectrum covers a range from 0.05um to about 50.0um while the THoT tool has a frequency spectrum that ranges 0.5um to 5,000um.

This raises the question of how can we verify the accuracy of the Rq measurement?

The answer is in the relationship of the Ra measurement to the Rq measurement. As will be seen in the following information, the ratio of the Ra to Rq should approach unity as the surface becomes more chaotic. And, conversely, on a theoretically absolutely flat surface, the ratio would, theoretically, approach infinity.

Therefore, if it can be demonstrated that the measurements return the proper ratios, we can consider that the Rq measurement is valid.

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This note considers a numerical factor which connects Ra, the arithmetic surface roughness with Rq, the root mean square roughness factor. The general definition of Ra is:

$$Ra = 1/L \int_0^L |Y(x)|dx$$

Eqn 1

and of Rq is:

$$Rq = \sqrt{1/L \int_0^L Y^2(x) \quad dx}$$

Eqn 2

Consider the special case where the surface roughness profile f(x) is a sine wave, as represented in Eqn 3.

$$Y(x) = b \sin x \qquad \text{Eqn 3}$$

so that the x-axis (or Y = 0) is the mean line by definition and "b" is the maximum amplitude of the sine wave or the greatest vertical distance from the highest point to the mean line.

For convenience in calculating Ra, consider only the first positive part of the sine curve in Eqn 3 so that the sampling length L is from x = 0 to x = π.

The Eqn 1 integral is:

$$Ra = \frac{B}{\pi} \left[-\cos x\right]_0^{\pi} = \frac{2b}{\pi}$$

Eqn 4

To obtain Rq, first square both sides of Eqn 2. It is only necessary to integrate Eqn2 over the sampling length from x = 0 to x = π, with the result:

$$(Rq)^2 = \frac{b^2}{\pi} \left[-\tfrac{1}{4} \sin (2x) + x/2\right]_0^{\pi} = \frac{b^2}{2}$$

Eqn 5

or:

$$Rq = \frac{b}{2^{1/2}}$$

Eqn 5A

Accordingly, the ratio of Rq to Ra is, for a sine wave surface profile (Eqn 3)

$$\frac{Rq}{Ra} = \frac{\pi^{1/2}}{2^{1/2}} = 1.1107$$

Eqn 6 (sine)

Repeating this same procedure, for a triangular saw-tooth profile of period 4L defined by the straight line equation Y = mx gives a similar result:

$$\frac{Rq}{Ra} = \frac{2}{3^{1/2}} = 1.1547$$

Eqn 7 (sine)

For a square wave profile of period 2L, defined by Y(x) = + b for 0 < x < L and Y(x) = - b for L < x < 2L,

$$Rq / Ra = 1.0000$$

Eqn 8 (square)

QED

A few readings on smooth, medium and rough surfaces shows that the preceding holds true.

| Measured Ra | Measured Rq | Calculated Ratio |
|---|---|---|
| 1.886 | 2.303 | 1.221 |
| 1.977 | 2.404 | 1.216 |
| 2.518 | 2.932 | 1.164 |
| 2.600 | 3.065 | 1.179 |
| 5.393 | 6.077 | 1.127 |
| 16.657 | 17.592 | 1.056 |